Low-Order Estimation of Dynamic MRI Sequences

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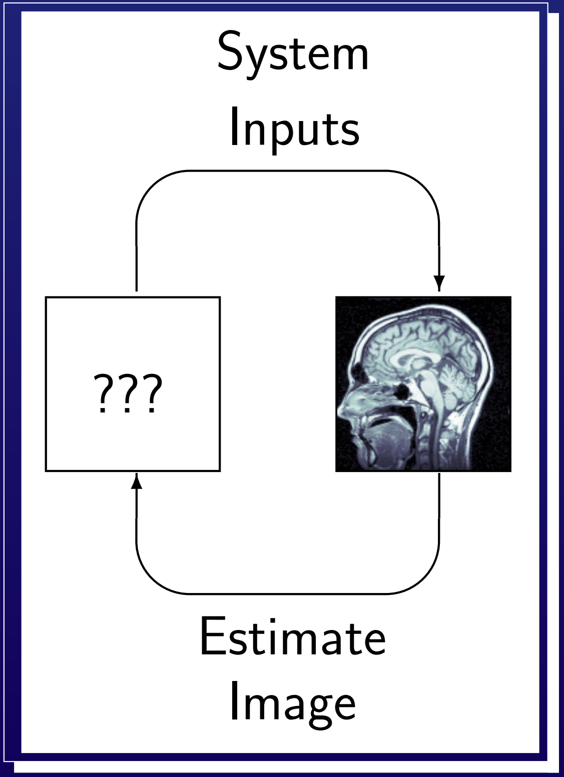
Problem Description

- **Goal:** *The efficient acquisition of dynamic MRI sequences.* Greater efficiency allows decreased image acquisition time while allowing one to maintain image quality including signal-to-noise ratio, field of view, and image resolution.
- **Methodology:**
  - Determine a cost function and parameterization that reflects the problem.
  - Apply cost function minimization techniques to find a solution.
- **Signal processing concepts related to this work** Adaptive Filtering, System Identification, and Subspace Tracking

Approach

- Common approach with previous methods (FK, SVD and kSVD, RIGR)
  - use previous image history to guide new estimates
  - Acquire subset of k-space data at time n
  - Use modeling to estimate image from output data and knowledge of the inputs
- Goal: find the "Best" ...
  - method to construct estimates.
  - choice of input vectors.

  ... through an appropriate cost function minimization problem.



Linear System Response Model

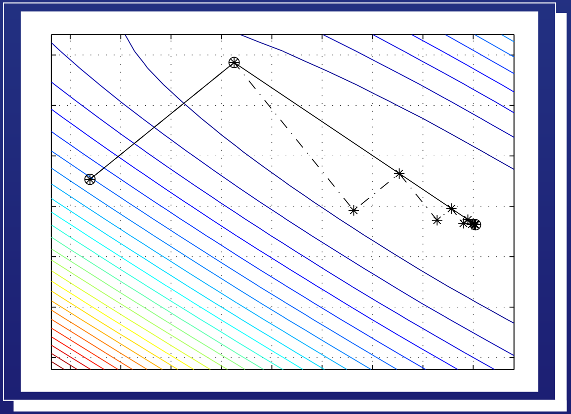
- **Criteria** (as demonstrated by Dr. Panych [1, 2]):
  - If one uses rf encoding and low flip angles,
  - one can approximate image acquisition equations to the first order.
  - Superposition then holds for the MR response.
- **Model Equation:**

$$Y_n = A_n X_n, \quad \begin{aligned} X_n &= \text{columns of input rf encoding vectors. } (N \times r) \\ A_n &= \text{Image Slice } (M \times N) \\ Y_n &= \text{Output (or measured) data } (M \times r) \end{aligned}$$

- **Conceptually:**
For this model, acquisition time is proportional to the number of rf encoding vectors used.

reduce number of inputs $\longrightarrow$ reduce acquisition time

Doubly Adaptive Strategy



- **Cost functions**
  - Output Error: $\mathcal{J}_n = \|\boldsymbol{Y}_n - \hat{\boldsymbol{A}}_n \boldsymbol{X}_n\|_F^2$
  - Estimate Error: $\mathcal{E}_n = \|\boldsymbol{A}_n - \hat{\boldsymbol{A}}_n\|_F^2$

- **Cost Function for Image Estimation**
  - Determine $\hat{\boldsymbol{A}}_n$ through minimization of

$$\mathcal{J}_n = \underset{\hat{\boldsymbol{A}}_n}{\mathbf{argmin}} \|\boldsymbol{Y}_n - \hat{\boldsymbol{A}}_n \boldsymbol{X}_n\|_F^2$$

  - Minimization problem is overdetermined, thus solution is non-unique.
  - Assume sequence is 'smoothly' varying.
  - Assume changes are "small", $||\boldsymbol{\Delta A}_n||_F^2 \ll ||\boldsymbol{A}_n||_F^2$.
  - We restrict the input matrix $\boldsymbol{X}_n$ s.t. the columns are orthonormal.

Image Reconstruction Methods (1)

- Given $\boldsymbol{Y}_n$ and $\boldsymbol{X}_n$, solving $\mathcal{J}_n = \|\boldsymbol{Y}_n - \hat{\boldsymbol{A}}_n \boldsymbol{X}_n\|_F^2 = \mathbf{0}$, gives a *Low-rank Reconstruction*

$$\hat{\boldsymbol{A}}_n = \boldsymbol{Y}_n \boldsymbol{X}_n^H (\boldsymbol{X}_n \boldsymbol{X}_n^H)^\dagger = \boldsymbol{Y}_n \boldsymbol{X}_n^H = \boldsymbol{A}_n \boldsymbol{X}_n \boldsymbol{X}_n^H$$

  Used in the SVD method, [2, Zientara, Panych, et. al.]

- Modeling image changes as $\hat{\boldsymbol{A}}_n = \boldsymbol{A}_0 + \boldsymbol{\alpha}_n$ and solving $\mathcal{J}_n = \mathbf{0}$ gives a *Keyhole Reconstruction*

$$\hat{\boldsymbol{A}}_n = \boldsymbol{A}_n \boldsymbol{X}_n \boldsymbol{X}_n^H + \boldsymbol{A}_0 (\boldsymbol{I} - \boldsymbol{X}_n \boldsymbol{X}_n^H)$$

  Used in the Fourier Keyhole method [3, van Vaals, Brummer, et. al.] and the Keyhole SVD method [4, Zientara, Panych, et. al.]

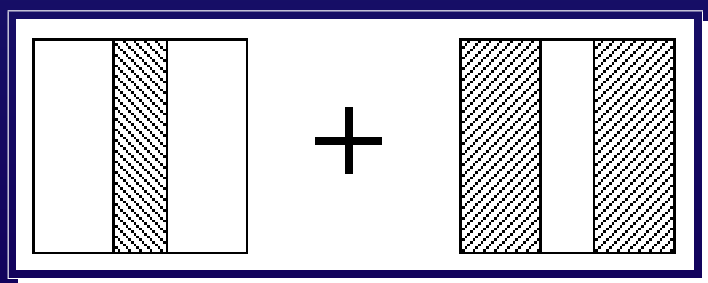


Image Reconstruction Methods (2)

- Modeling changes as $\hat{A}_n = \hat{A}_{n-1} + \beta_n$ and solving $\mathcal{J}_n = 0$ gives an *Adaptive Framework Reconstruction*

$$\hat{A}_n = Y_n X_n^H + \hat{A}_{n-1}(I - X_n X_n^H).$$

  Rewriting this expression as

$$\hat{A}_n = \hat{A}_{n-1} + \left(Y_n - \hat{A}_{n-1} X_n\right) X_n^H$$

  one can recognize similarities with the familiar LMS adaptive filter update equation.

- Estimate error at time n,

$$\mathcal{E}_n = \|A_n - \hat{A}_n\|_F^2 = \|(A_n - \hat{A}_{n-1})(I - X_n X_n^H)\|_F^2,$$

  can guide selection of next input vectors.

- Optimal inputs are thus the right singular vectors of the difference between the most recent estimate and the *next* image.
  (*Note:* non-realizable)

Dynamic Determination of New Inputs

- **Linear Prediction**
  A_{n+1} is not known at time n.
  - Introduce a linear predicted estimate, $\tilde{A}_{n+1} = \Sigma_{i=0}^{l} c_i \breve{A}_{n-i}$, to replace A_{n+1} in $\mathcal{E}_{n+1}$.
  - Determine new input vectors via dominant right singular vectors of

$$(\tilde{A}_{n+1} - \hat{A}_n).$$

- **Subspace Trap**
  Estimates tend to be biased towards previous inputs because they can only reflect image changes supported by the input subspace, $X_n X_n^H$
  - For example, with $\tilde{A}_{n+1} = \breve{A}_n + (\breve{A}_n - \breve{A}_{n-1})$, and the adaptive framework estimate, $\breve{A}_n = Y_n X_n^H + \hat{A}_{n-1}(I - X_n X_n^H)$,

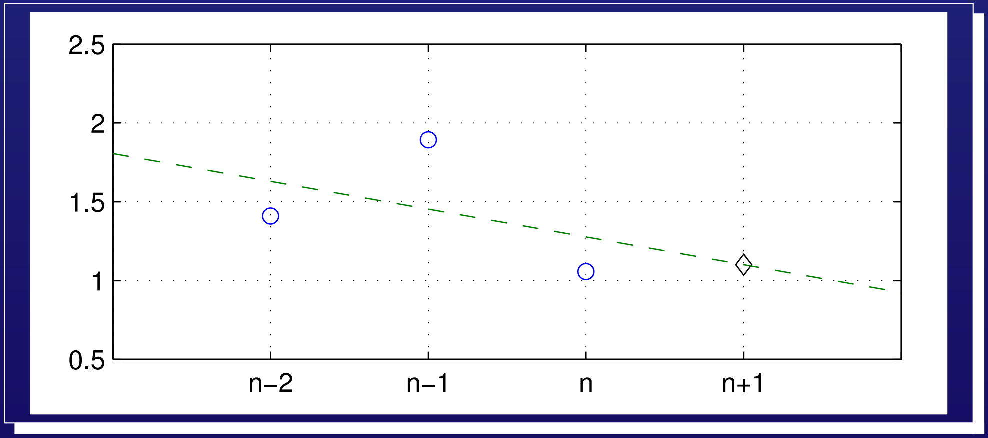
$$(\tilde{A}_{n+1} - \hat{A}_n) = (A_n - \hat{A}_{n-1}) X_n X_n^H$$

Linear Temporal Prediction Solution

- **Simple Solution**: pixel-by-pixel temporal extrapolation of new point from three previous points

$$\tilde{A}_{n+1} = \frac{4}{3}(Y_n X_n^H) + \frac{1}{3}(Y_{n-1} X_{n-1}^H) - \frac{2}{3}(Y_{n-2} X_{n-2}^H)$$

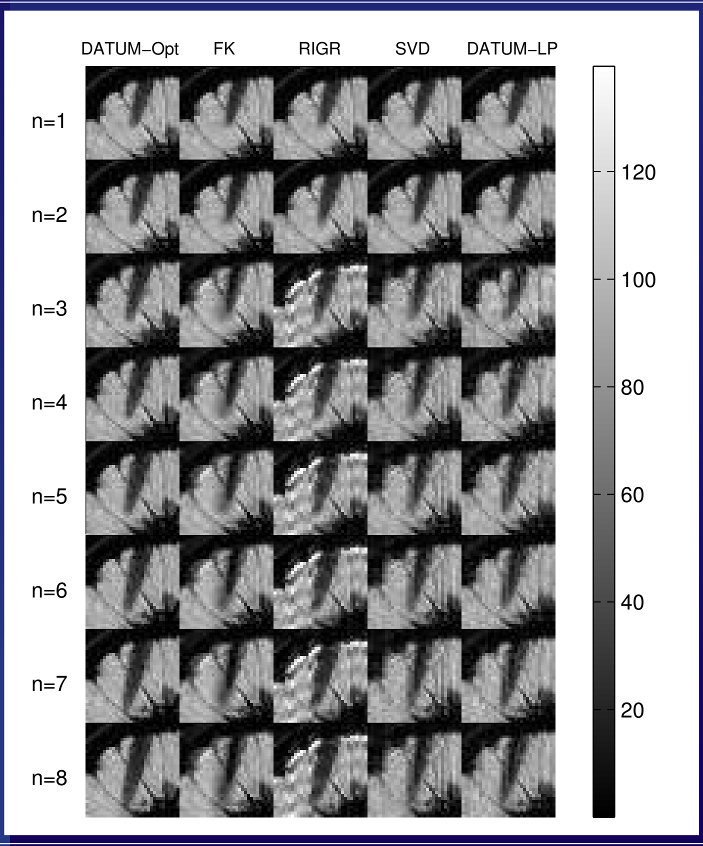
using the *low-rank estimate*.

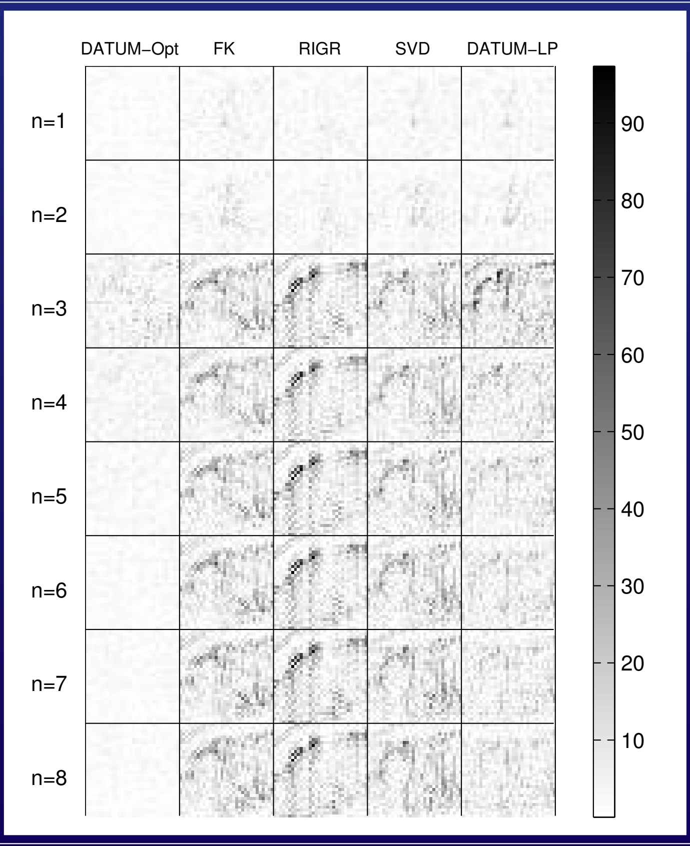


- **Open Questions:**
  - Optimal choice of inputs for linear predictor, denoted $\breve{A}_n$
  - Appropriate predictive models (application dependent?)

Method Comparison

Fourier Keyhole (FK) • Keyhole SVD (SVD) • RIGR
Doubly Adaptive Temporal Update Method-Linear Prediction (DATUM-LP)
Theoretical Optimum (DATUM-Opt)





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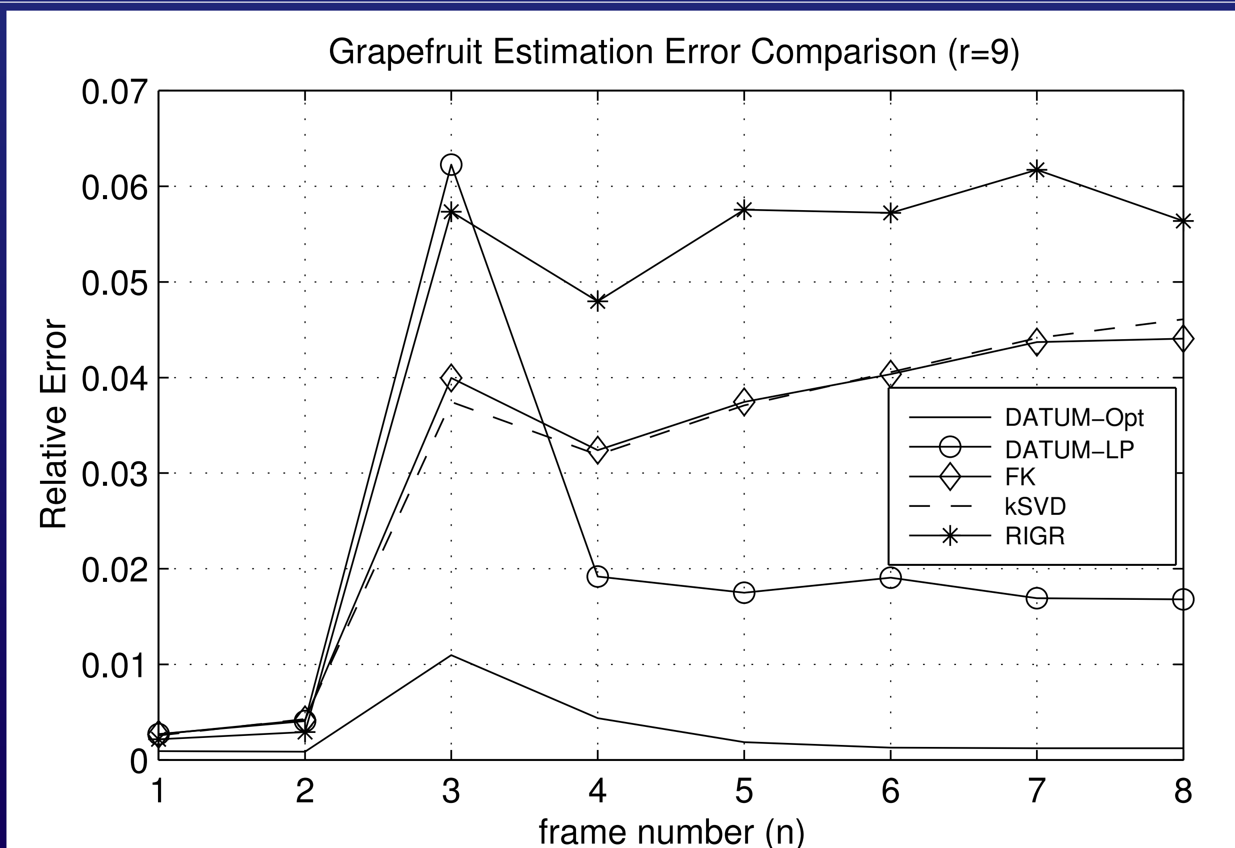

References

[1] L. P. Panych, G. P. Zientara, and F. A. Jolesz, "MR image encoding by spatially selective rf excitation: An analysis using linear response models," *Int. J. Imaging Syst. Technol.*, vol. 10, no. 2, pp. 143–150, 1999.

[2] G. P. Zientara, L. P. Panych, and F. A. Jolesz, "Dynamically adaptive MRI with encoding by singular value decomposition," *Magn. Reson. in Med.*, vol. 32, pp. 268–274, 1994.

[3] J. van Vaals, M. E. Brummer, and et. al., "Keyhole method for accelerating imaging of a contrast agent uptake," *J. Magn. Reson. Imag.*, vol. 3, no. 4, pp. 671–675, 1993.

[4] G. P. Zientara, L. P. Panych, and F. A. Jolesz, "Keyhole SVD encoded MRI," in *Proceedings SMR, 2nd Annual Meeting*, (San Francisco), p. 778, 1994.