Pull-Push Level Sets: A new term to encode prior knowledge for the segmentation of teeth images

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ABSTRACT

This paper presents a novel level set method for contour detection in multiple-object scenarios applied to the segmentation of teeth images. Teeth segmentation from 2D images of dental plaster cast models is a difficult problem because it is necessary to independently segment several objects that have very badly defined borders between them. Current methods for contour detection which only employ image information cannot successfully segment such structures. Being therefore necessary to use prior knowledge about the problem domain, current approaches in the literature are limited to the extraction of shape information of individual objects, whereas the key factor in such a problem are the relative positions of the different objects composing the anatomical structure. Therefore, we propose a novel method for introducing such information into a level set framework. This results in a new energy term which can be explained as a regional term that takes into account the relative positions of the different objects, and consequently creates an attraction or repulsion force that favors a determined configuration. The proposed method is compared with balloon and GVF snakes, as well as with the Geodesic Active Regions model, showing accurate results.

Keywords: Level Sets, Segmentation, Prior Information, Teeth

1. INTRODUCTION

In accordance with the increasing relevance of medical imaging in the different medical areas, dental practices have also taken advantage of different medical image modalities, and image processing techniques can help improve all the phases of dental treatment. Specifically, this work deals with diagnosis and treatment planning in orthodontic practice.

In orthodontics, the detection and correction of malocclusions and other dental abnormalities[1] requires accurate information about the shape and position of teeth of both arches.

The procedure for manually obtaining the information required for the diagnosis and treatment planning is time consuming and costly. So, the automation of this process is a desirable goal.[2] In principle, this could be done by studing a 2-D image of a dental plaster cast model of each dental arch and extracting each tooth contour. An example of such an image can be seen in Figure 1.

This paper focuses on the problem of teeth contour segmentation from arch cast model images using deformable models. Three different methods, with an increasing refinement, are described and analyzed. First, a method using traditional active contours, or snakes,[3] is proposed, starting from initial contours inside each tooth. This approach showed weaknesses related to well-known problems as badly defined boundaries and the inability of snakes to deal with changes in topology. So, in order to overcome these drawbacks, a second method was designed, based on the use of geodesic active regions in a level set framework.[4] However, new problems

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Figure 1. Sample image of a plaster cast of a dental arch.

dealing with merging of different contours into a single one (which is an undesired effect) appeared. Consequently, we developed a modified version of geodesic active regions, which is the main contribution of this paper, that introduces a new force that takes into account interactions between the different objects so that the final segmentation is in accordance with prior knowledge about the anatomical structure of the dental arch.

The structure of the paper is as follows. In section 2 an algorithm for the determination of the initial contours is briefly described. In sections 3, 4 and 5, following the logical reasoning mentioned earlier, all three methods will be described, emphasizing the original contributions, and results will be given and discussed. Finally, we will draw some conclusions and future work directions.

2. DETERMINATION OF INITIAL CONTOURS

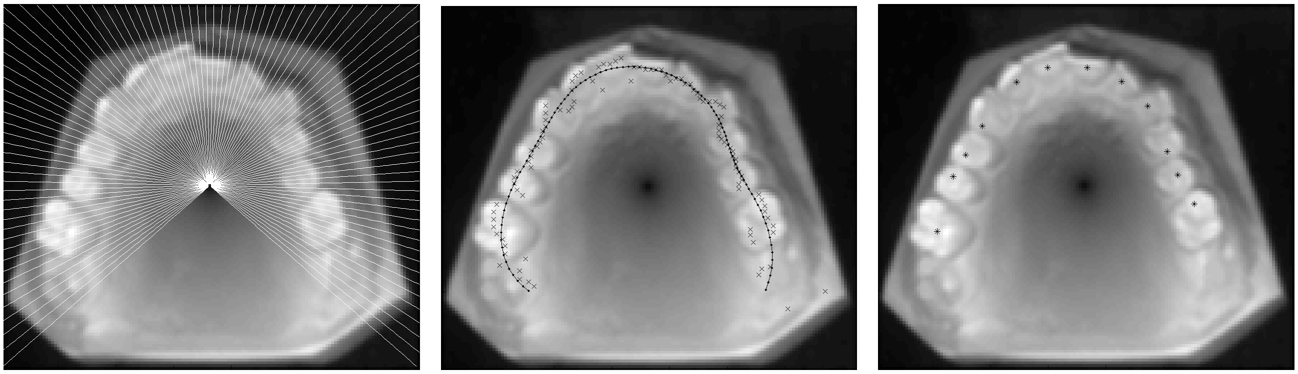
Initial contours are found exploiting the prior knowledge about the general structure of the dental arch. The process is ilustrated in Figure 2, and works as follows:

- First, starting from the center of the image, rays are traced radially, and one-dimensional intensity profiles are extracted along these rays (Figure 2(a)).
- Then, each section is analyzed and a point is located where the ray crosses the dental line. This way, a set of points is obtained which approximately follows the dental line (see Figure 2(b)). In order to remove possible abnormalities, the line is smoothed using a few terms of a Discrete Fourier Series.
- Finally, the dental line determines a longitudinal section crossing all teeth, which is analyzed to find peaks and valleys that correspond to the teeth and inter-teeth gaps. This way, a point is located inside each tooth (Figure 2(c)).

Efforts have been made in order to make this algorithm as robust as possible. However, the user must supervise the process and reposition the points which could have been misplaced. Once the points (seeds) have been found, initial contours are defined simply by creating a small contour around each seed.

3. SEGMENTATION WITH SNAKES

Active contours or snakes were first introduced in,[3] and the model was later extended in[5] with the introduction of an inflation force. Snakes are energy minimizing curves usually driven by internal, image and external energy terms. Active contours have successfully been employed for the segmentation of anatomical structures, as in[6–8]



(a) (b) (c)

Figure 2. (a) Radial rays traced from the image center. (b) Dental line crossing points and smoothed dental line (solid line). (c) Final points located inside each tooth.

Usually, an inflation energy term is used[5] because the image energy alone cannot attract the snake when it is far from the boundaries. However, because of stability reasons, a *Gradient Vector Flow* force[9] was used instead, increasing the capture range of the snakes and making unnecessary the inflation force.

Using the active contours described before, it is possible to employ a snake for the segmentation of each tooth in the dental arch images. However, appropiate parameters for the GVF force calculation are very dependent, not only on the particular image but also on the particular tooth being segmented. To address this problem an automatic supervisor was constructed that tries several parameters and decides, for each tooth, the optimal segmentation.

The described method was tested on several images and, although it performs accurately for many teeth, in other ones poor results are obtained, as can be seen in Figure 3. Problems are caused, first, because certain areas in the images are affected by undesired shadows, thus resulting in badly defined boundaries. This causes the active contours to leak away from the correct edges. Second, as snakes cannot naturally handle topology changes it is necessary to employ a single snake for each tooth, and overlapping cannot be easily avoided.

In order to overcome the stated problems, geodesic active regions were tried. In the next section, we justify this election and describe the method that was proposed and implemented.

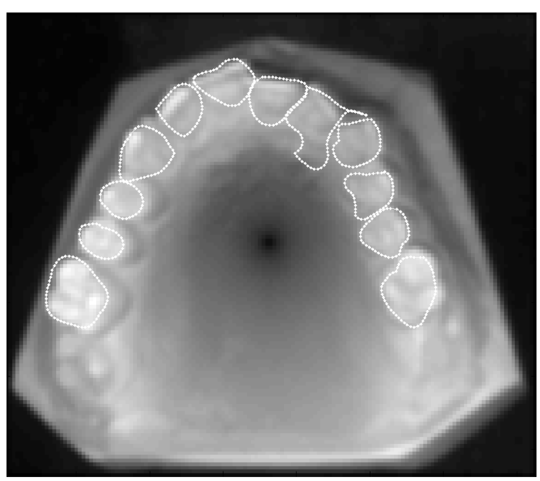


Figure 3. Teeth segmentation results using GVF snakes.

4. SEGMENTATION WITH GEODESIC ACTIVE REGIONS

Snakes, or parametric contours, cannot deal naturally with topology changes. To solve this problem, level set-based segmentation was introduced.[10] This approach is based on the original work by by Osher and Sethian to model propagating interfaces with curvature-dependent speeds.[11, 12] The level set aproach to image segmentation has gained much relevance lately because of the good properties of this method, and has been successfully applied to image segmentation in 2D and 3D.[13–15]

From another perspective, image segmentation methods have traditionally been divided in two groups: boundary and region based. Both kinds of information have been considered in a level set framework.[4, 10, 16–19] The last work, by Paragios *et al.*, is especially interesting as it allows the integration of both approaches. Starting from the *geodesic active contour* model,[16] region information was introduced and so the *geodesic active region* model was proposed. In this model, both boundary and region information are employed in a level set framework. Specifically, let us suppose a two-class problem (segmentation of a certain structure of interest in a background). Then, the segmenting curve will be determined as the zero-level set of a surface function ϕ, the evolution of which, for a pixel $\mathbf{u} = (x, y)$, will be given by:

$$\frac{\partial \phi}{\partial t}(\mathbf{u}) = \alpha r(\mathbf{u})|\nabla \phi(\mathbf{u})| + (1-\alpha)(b(\mathbf{u})\mathcal{K}(\mathbf{u}) + \nabla b(\mathbf{u}) \cdot \frac{\nabla \phi(\mathbf{u})}{|\nabla \phi(\mathbf{u})|})|\nabla \phi(\mathbf{u})| \tag{1}$$

where α is a constant that weighs the first and second terms of the sum ($0 \leq \alpha \leq 1$), $\mathcal{K}$ is the boundary curvature[12] and $r(\mathbf{u})$ and $b(\mathbf{u})$ are the region and boundary functions, respectively. The first term weighted, by α, is called the "region term", and is controlled by the region function $r(\mathbf{u})$, which aims at shrinking or expanding the curve according to some kind of region information (see[4] for details). As with regard to the boundary force, which is the term affected by $1-\alpha$, it is controlled by the boundary function $b(\mathbf{u})$ which captures the boundary information in the image and returns minima values for the boundary pixels.

Compared to the previously described snakes, geodesic active regions benefit from having no problem with topology, since geodesic active regions make use of the level set theory. Besides, they are more robust because they use not only boundary but also region information.
The described approach was employed in this work to extract teeth contours. Based on the evolution expression given in Eq. 1, both boundary and region information were employed. Instead of the region force proposed in,[4] for which great deals of data would be needed, an alternative expression was employed:

$$r(\mathbf{u}) = \begin{cases} -1 & \text{if } \eta - k\sigma \leq I(\mathbf{u}) \leq \eta + k\sigma \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where η and σ are the mean and variance of the intensity values inside region A, and k is a possitive constant. This way, the region function $r(\mathbf{u})$ favors regions whose pixel intensities are similar to those which are already considered as being part of the segmented region A. With regard to the boundary term, a boundary function based on the image gradient was employed.[15]

The described method was tested on several images. Some results are shown in Figure 4. It can be seen that different contours have merged, as the boundaries between teeth are not well defined. To overcome this problem, in the next section we introduce and describe the so-called *Pull-Push Level Sets*. This model is based on the geodesic active regions, and incorporates prior knowledge about the general shape of the structure (composed of several objects) to be segmented.

5. PULL-PUSH LEVEL SETS

Apart from boundary and region information, still more information can be incorporated to improve the segmentation. Sometimes, as in this case, shape information can be useful for the segmentation of the structures of interest.

Shape information has successfully been employed in image segmentation algorithms in the literature. In this direction, Cootes *et al.* developed ASMs (*Active Shape Models*).[20] More recently, in a level set framework,

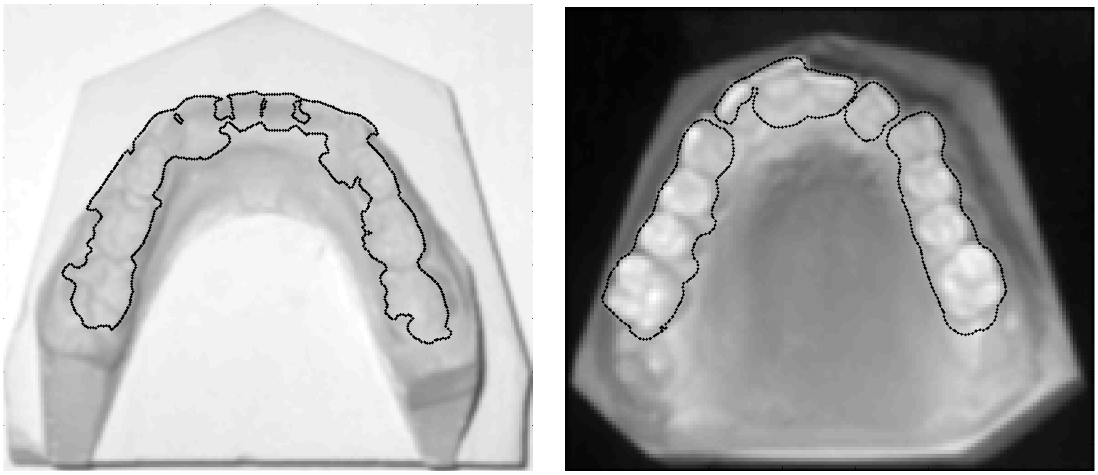


Figure 4. Results of the segmentation of teeth images using geodesic active regions.

Leventon *et al.* introduced shape information by means of creating a curvature profile from a training set.[21] In,[22] principal component analysis is applied to create a representation of the training data in terms of a signed distance function. This work could be considered as a level set extension for ASMs.

Nevertheless, all these methods were designed for the description of a single object's shape, rather than the relative positions of several objects in a scene. So, in order to incorporate to our segmentation scheme prior information about the relative positions of the several items to be segmented, the *Pull-Push Level Sets* are here proposed. In a geodesic active region framework, the main idea is to introduce a new term so that each object in the scene will pull (attract) or push (repel) the others thus favoring the most probable configurations.

Mathematically, the new term is encoded as an attraction-repulsion force field which adds to the region term in the geodesic active region model. This field, for a given pixel $\mathbf{u}$, depends on the relative positions of the two nearest objects. The region function $r(\mathbf{u})$ then becomes:

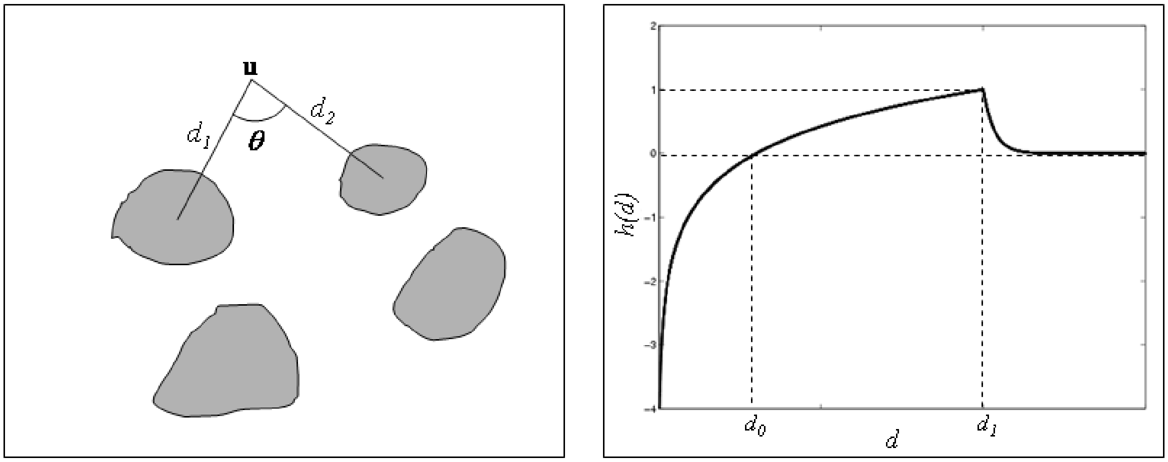
$$r'(\mathbf{u}) = r(\mathbf{u}) - f(\theta) \cdot h(d) \tag{3}$$

where $r(\mathbf{u})$ is the region function as appearing in Eq. 1 (especifically, in this work $r(\mathbf{u})$ was as defined in Eq. 2). The new term can be considered as a pull-push force field that, for a given pixel $\mathbf{u}$, depends on the relative positions of the two nearest objects. Let us consider the situation depicted in Figure 5 (a), where, at iteration step i, there are several non-connected segmented objects (an object will be defined as a connected region belonging to class A, or, equivalently, having $\phi(\mathbf{u}) < 0$). Then, for every pixel $\mathbf{u}$, two segments are traced to the centers of the two nearest objects, and θ is defined as the angle between them ($0 \leq \theta \leq \pi$). d_1 and d_2 are the distances from pixel $\mathbf{u}$ to both object centers, and d is defined as $d = \max\{d_1, d_2\}$. $f(\theta)$ can be now defined as follows:

$$f(\theta) = e^{\beta(\frac{\theta}{\pi} - 1)} \tag{4}$$

where β can be adjusted to obtain a smoother or steeper variation. As with regard to function $h(d)$, this function behaves as plotted in Figure 5 (b).

Globally, this new term acts as an attraction force between two objects if they are not too far away (this pull-force decreases exponentially if the distance is bigger than d_1, see Figure 5). However, if the two objects come very close to each other, then the pull-force becomes a push-force, repelling the two objects and thus preventing them from merging (when the distance is smaller than d_0, see Figure 5). Note that while $h(d)$ is responsible for the pull-push behaviour of the force field, the term $f(\theta)$ is designed in order to attenuate the force field for pixels that do not lie between two near objects. To illustrate this, let us consider the situation depicted in Figure 6. If we assume that points A and B are not far away, then, approximately, $d_1 \approx d_3$ and $d_2 \approx d_4$. If there were no



(a) (b)

Figure 5. (a) Graphical interpretation of variables θ, d_1 and d_2. (b) Graphical representation of function $h(d)$.

$f(\theta)$ term, then both pixels would have similar push-pull force field values and, consequently, the contours would expand in all directions (i.e. the attraction and repulsion forces would simply become inflation and deflation ones). However, $f(\theta)$ prevents this from happening because, as $\theta_B < \theta_A$, $f(\theta_B) < f(\theta_A)$, and so the pixels that lie near the line that joins the two objects will be more affected. This way, the overall force actually behaves as a pull-push force field.

Experiments have been carried out that show the suitability of the proposed model for the segmentation of teeth images. Some examples can be seen in Figure 7. Indeed, the proposed method shows a better overall performance than the approaches discussed earlier, mainly because the pull-push force drives the segmentation close to the real boundaries. Then, region and boundary forces guide the contours towards the most prominent features.

6. CONCLUSIONS AND FUTURE WORK

In this work, three different segmentation methods have been proposed with increasing refinement. First, active contours or snakes were employed with a GVF force, but results showed some inherent limitations of this approach. The use of geodesic active regions overcomes some of these drawbacks. However, region information

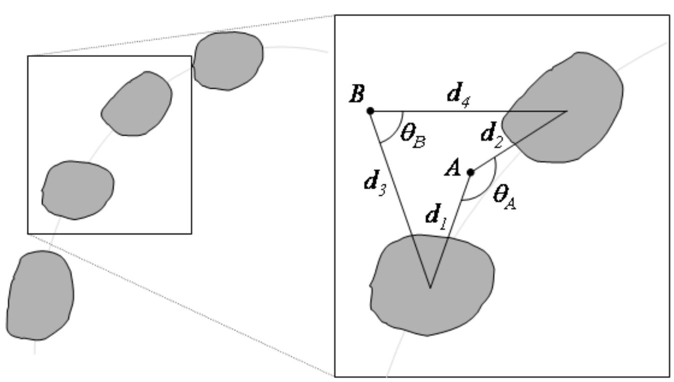


Figure 6. Relative distances and angles for two pixels A and B.

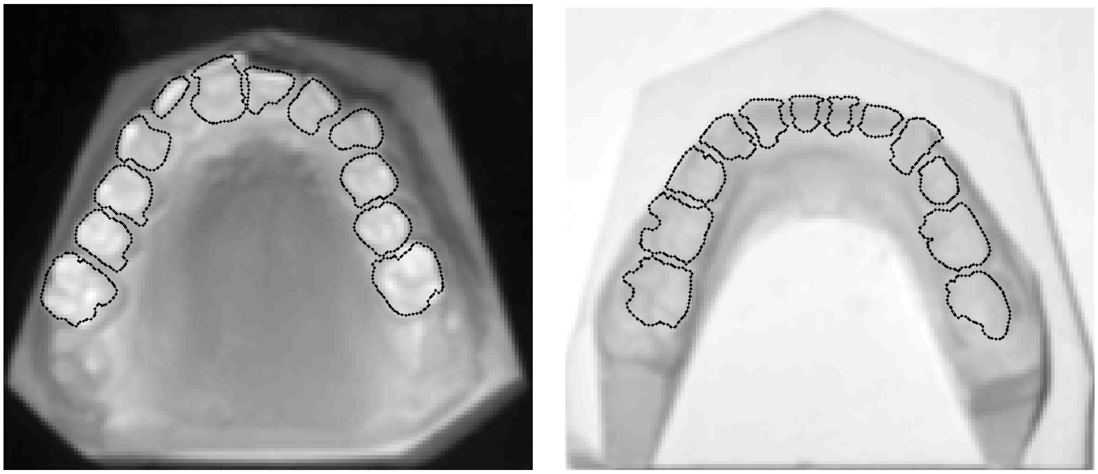


Figure 7. Segmentation results using pull-push level sets.

was not able to successfully guide the segmentation process, and problems appeared related to the multi-object nature of the scene.

Thus, a new term was proposed that precisely aims at encoding prior knowledge regarding to the relative positions of the different objects in the scene. This new term was developed in a geodesic active region framework in order to maintain the advantageous properties of this model.

It is important to note that efforts have been made in this work in order to achieve a robust segmentation method. Inevitably, this implies a decrease in the accuracy of results, and so a compromise has to be made between robustness and accuracy.

Although further investigation must be made in order to refine this method, this work opens up a new direction by adding object interaction to the geodesic active regions model. Future work will also deal with the generalization of the pull-push force to adapt to different situations.

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