MAXIMUM LIKELIHOOD CONTOUR ESTIMATION USING BETA-STATISTICS IN ULTRASOUND IMAGES

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Abstract

In this paper we address the problem of detecting the contour of objects in speckled ultrasound images. A prior hypothesis is made on the first order statistics of the image intensity that is related to the shape of the objects: the mean and variance of the images can be non-stationary, but they should be invariant to the normalized distance from a particular point in the image to the contour center. Furthermore, the first order statistics are modeled with a Beta distribution. The choice for that distribution is also validated with some experiments and seems to be more realistic than the log-compressed Rayleigh distribution reported in the literature for ultrasound images. These hypotheses can be validated for particular images by means of the hypotheses tests that we also propose in the paper. The algorithm is initially tuned to a particular image class by estimating the model parameters from a prototypical image of that class. Once our model is set, the contour is estimated in test images using the maximum likelihood (ML) criterion. The ML estimator is adjusted so as to give some means of contour regularization.

1 Introduction

The problem of image segmentation is present in any image analysis application. Despite of being ubiquitous, no general solution exists. Classical approaches [2], classify the segmentation techniques as discontinuity detection, boundary detection, thresholding and region-oriented. Usually these low level operations never reach an acceptable level of accuracy in complicated situations.

In most cases, high level techniques which exploit prior knowledge about the shapes of the structures to be segmented out are the only feasible solution. Snakes [3], or more generally, active contours and active rays [1, 4] are an example of those.

For particular applications, the more knowledge is incorporated into the segmentation framework, the higher the probability of the algorithm to be successful. This is the approach we make in the paper to solve the problem of semiautomatically locating the contours of objects in ultrasound images. As it is well known, this type of images suffer from speckle, have a low signal to noise ratio (i.e., contours are not clearly defined), and even an expert may have trouble interpreting ultrasound data not directly scanned by him/her.

To achieve our goal, we make use of a prior hypothesis about means and variances of the image pixels that is related to the position of the contour. The problem of contour detection is therefore posed as a problem of estimation. Both the mean and the variance functions are calculated from a set of training images, where the objects are placed at canonical positions; this information will be introduced in the image model to segment a test image. We assume that, with respect to the test image, the center of the object is known, either by previous coarse segmentation methods or by manually indicating one landmark on the images. First order statistics of the ultrasound images are approximated with a Beta distribution. This later hypothesis is also verified and compared to log-compressed Rayleigh, and it has the additional advantage of being computational manageable.

2 Contour Model

Figure 1 illustrates both the concepts and the notation that will be used throughout the paper. The contour center will be denoted by \((C_x, C_y)\). The rest of the variables will be referenced to this point. From that center we divide the image into \(J\) angular sectors (the boundaries of which are represented in solid line
in figure 1). The set of pixels within the j-th angular sector will be denoted by $\epsilon(j)$. Every angular sector is split into two equal sectors by drawing a spoke from the center of the contour outwards (dashed lines in figure 1). We will call $\theta_j$ the phase of each spoke. Those $\theta_j$ are not angularly equispaced necessarily. Each spoke is discretized into $K$ values $\theta_k$, with $k = 1, \ldots, K$ for each $j$. The set of pixels belonging to $\epsilon(j)$ and closest to the point $r_k$ will be denoted by $\delta(k, j)$. The (unknown) position of the true contour for every ray, $R(j)$, will be picked from one of the $r_k$. In practice, the maximum value of $r_k$ is given by the distance of the farthest pixel to the center, and the minimum to one twentieth of the maximum. The $r_k$ will be typically equispaced.

![Figure 1. Polar discretization of the image.](image1)

We assume that $R = [R(1) \ldots R(J)]^T$, is a monovalued vector function: the objects are either convex or star-like and no severe concavities are present.

### 3 Image Model

We suppose that the original image $X(m, n)$ has $M \times N$ pixels. The image is an intensity image, with gray levels $0 \leq X(m, n) \leq 1$, where 0 means black and 1 white.

The ultrasound image formation process stems from the detection of tissue backscatterers. The backscattered ultrasonic signal can be expressed as the sum of the phasors from the individuals scatterers. When the number of scatterers is large within each resolution cell, the probability density function of the envelope of the signal can be modeled by a Rayleigh distribution [6]. On the other hand, if the number of scatterers in the resolution cell is low the Rayleigh statistics is a poor approximation. In these cases, a K distribution may be used to model the signal envelope [5].

Clinical ultrasound systems, actually, use a logarithmic compression in order to reduce the dynamic range of the envelope signal. This nonlinear processing modifies the signal statistics. Although a closed expression for the log-transformed K distribution is yet to be derived, the log transformed Rayleigh distribution can be easily worked out.

When dealing with real images, deviations from the log-transformed Rayleigh distribution are frequently observed. To overcome this situation, in this paper we have used a Beta distribution to model the log-compressed ultrasound image. Beta distributions approximate bounded empirical distributions fairly well, and this fact has been observed experimentally. Figure 2 shows the histogram of a sketched sample from an ultrasound of (a) a real tissue and (b) a phantom which will be used later. In both cases two theoretical probability density functions corresponding to log-transform Rayleigh (dashed line) and (b) Beta distribution (solid line) have been superimposed. The parameters of the distributions have been estimated by the method of moments.

![Figure 2. Distribution fitting of (a) phantom and (b) real speckled sample.](image2)
the image pixel to the object center normalized by the distance to the object contour on the angular sector of the pixel to the object center, that is

\[ \eta(m,n) = \eta \left( \frac{r_k}{R(j)} \right) \Longleftrightarrow (m,n) \in \delta(k,j) \]  
(1)

\[ \sigma^2(m,n) = \sigma^2 \left( \frac{r_k}{R(j)} \right) \Longleftrightarrow (m,n) \in \delta(k,j) \]  
(2)

If we interpolate those functions we can write \( \eta(\alpha) \) for the mean and \( \sigma^2(\alpha) \) for the variance. The dependence with respect to \( j \) is through the knowledge of the true contour \( R(j) \). Therefore, our model is affine invariant. From this, it is clear that \( \eta(1) \) and \( \sigma^2(1) \) are the mean and the variance of the pixels on and nearby the contour.

In order to estimate the functions \( \eta(\alpha) \) and \( \sigma^2(\alpha) \) the procedure is as follows: from a prototypical image (or images) that belongs to the class of images of interest, we manually outline the desired contour \( R \). We also set the values of the parameters \( J \) and \( K \), that is, the number of sectors and the number of points along a spoke.

By means of a landmark (or any other method), we obtain the center \( (C_x, C_y) \). The only piece of information that is still missing to calculate the above-mentioned functions \( \eta(\alpha) \) and \( \sigma^2(\alpha) \) is how the image pixels in the prototypical image(s) will be arranged for the calculation. This is now explained in detail.

Specifically, and as a first step, the two functions will only be calculated in a grid of points of the \( \alpha \)-domain. We use \( p \) points \( \alpha_p \) \( (p = 0, \ldots, P - 1) \). Then two mask images, \( M_1 \) and \( M_2 \), are calculated as

\[ M_1(m,n) = j \Longleftrightarrow (m,n) \in \epsilon(j) \]  
(3)

\[ M_2(m,n) = \frac{(m - C_x)^2 + (n - C_y)^2}{R[M_1(m,n)]} \]  
(4)

\( M_1(m,n) \) indicates the number of the sector that the pixel \( (m,n) \) belongs to, \( M_2(m,n) \) is the distance of the pixel \( (m,n) \) to the center \( (C_x, C_y) \) normalized by \( R(j) \). The \( \alpha_p \) points will be determined by

\[ \alpha_p = \frac{1}{P} \max_{m,n} M_2(m,n) \]  
(5)

Pixel at coordinates \( (m,n) \) will be assigned to \( \alpha_p \) if it falls on \( \mu(p) \), the region of influence of \( \alpha_p \),

\[ (m,n) \in \mu(p) \Longleftrightarrow |M_2(m,n) - \alpha_p| < |M_2(m,n) - \alpha_q| \quad \forall q \neq p. \]  
(6)

We also define a new region \( \gamma(p,j) \) with the expression

\[ (m,n) \in \gamma(p,j) \Longleftrightarrow (m,n) \in \mu(p) \cap (m,n) \in \epsilon(j). \]  
(7)

We define a new image \( X_{\alpha_p} \): the local mean of \( X \) as

\[ X_{\alpha_p}(m,n) = \frac{1}{L} \sum_{a=-\frac{L-1}{2}}^{\frac{L-1}{2}} \sum_{b=-\frac{L-1}{2}}^{\frac{L-1}{2}} X(m + a, n + b), \]  
(8)

where \( L \) is a given odd integer. We also define \( X_{\alpha_p} \): the local variance of \( X \) as

\[ X_{\sigma^2}(m,n) = \frac{1}{L} \sum_{a=-\frac{L-1}{2}}^{\frac{L-1}{2}} \sum_{b=-\frac{L-1}{2}}^{\frac{L-1}{2}} [X(m + a, n + b) - X_{\alpha_p}(m + a, n + b)]^2. \]  
(9)

The mean and variance for each sector \( j \) is

\[ \eta_j(\alpha_p) = \frac{\sum_{(m,n) \in \gamma(p,j)} X_{\alpha_p}(m,n)}{\vert \gamma(p,j) \vert} \]  
(10)

\[ \sigma^2_j(\alpha_p) = \frac{\sum_{(m,n) \in \gamma(p,j)} [X_{\alpha_p}(m,n) - \eta_j(\alpha_p)]^2}{\vert \gamma(p,j) \vert}, \]  
(11)

where \( \vert \gamma(p,j) \vert \) is the number of pixels in the set \( \gamma(p,j) \).

The values for \( \eta(\alpha_p) \) and \( \sigma^2(\alpha_p) \) are determined by

\[ \eta(\alpha_p) = \frac{1}{J} \sum_{j=1}^{J} \eta_j(\alpha_p) \]  
(12)

\[ \sigma^2(\alpha_p) = \frac{1}{J} \sum_{j=1}^{J} \sigma^2_j(\alpha_p). \]  
(13)

Finally those parameters are converted into continuous functions by doing aperiodic parametric cubic spline interpolation to finally get the desired functions \( \eta(\alpha) \) and \( \sigma^2(\alpha) \).

4 Maximum Likelihood Contour Estimation

Once the parameters of our model have been determined, we are ready to estimate contours for new images of the same class as the ones used for training purposes. As previously said, we use a Beta distribution as the intensity model. Its associated probability density function is given by

\[ f(x) = \left\{ \begin{array}{ll} \frac{x^{a-1}(1-x)^{b-1}}{B(a_1,a_2)} & 0 < x < 1 \\ 0 & \text{otherwise} \end{array} \right. \]  
(14)

where \( B(a_1,a_2) \) is the Beta integral given by

\[ B(a_1,a_2) = \int_0^1 t^{a_1-1}(1-t)^{a_2-1} dt. \]  
(15)
with $\alpha_1 > 0$ and $\alpha_2 > 0$ the shape parameters of the distribution. It is well-known that the mean $\eta$ and the variance are given by

$$\eta = \frac{\alpha_2}{\alpha_1 + \alpha_2} \quad (16)$$

$$\sigma^2 = \frac{\alpha_2}{(\alpha_1 + \alpha_2)(\alpha_1 + \alpha_2 + 1)} \quad (17)$$

After some algebra, the shape parameters $\alpha_1$ and $\alpha_2$ can be determined by the method of moments as

$$\alpha_1 = \frac{\sigma^2 - \eta^2 - \eta^2}{\sigma^2} \quad (18)$$

$$\alpha_2 = \frac{\sigma^2 - \eta^2 - \eta^2 (1 - \eta)}{\sigma^2} \quad (19)$$

Thus, we can easily obtain the shape parameters as functions $\alpha_1(\alpha)$ and $\alpha_2(\alpha)$ directly from the knowledge of $\eta(\alpha)$ and $\sigma^2(\alpha)$, using the equations above.

From a given image $X$ of size $M \times N$ we set the values for $J$ and $K$. These values could be different from those used in the model estimation procedure. Assuming we know the position of the contour center $(C_x, C_y)$, and according to our image model, we can state that the joint density function of $X$ conditioned to the contour $R$, can be phrased as

$$f(Z/R) = \prod_{j=1}^{J} \prod_{(m,n) \in \epsilon(j)} X(m,n)^{\alpha_1(\theta(m,n,\hat{R}(j))) - 1} \times \frac{1 - X(m,n)^{(\theta(m,n,\hat{R}(j))) - 1}}{B(\alpha_1(\theta(m,n,\hat{R}(j))), \alpha_2(\theta(m,n,\hat{R}(j))))} \quad (20)$$

where $\theta(m,n,\phi)$ is a deterministic function defined as:

$$\theta(m,n,\phi) = \sqrt{(m - C_x)^2 + (n - C_y)^2} / \phi \quad (21)$$

We now define the following sets

$$\kappa(j) = \epsilon(j - 1) \cup \epsilon(j) \cup \epsilon(j + 1) \quad 2 \leq j \leq J - 1$$

$$\kappa(1) = \epsilon(J) \cup \epsilon(1) \cup \epsilon(2)$$

$$\kappa(J) = \epsilon(J - 1) \cup \epsilon(J) \cup \epsilon(1) \quad (22)$$

and make the following approximations

$$R(j - 1) \approx R(j + 1) \approx R(j) \quad 2 \leq j \leq J - 1$$

$$R(J) \approx R(2) \approx R(1)$$

$$R(J - 1) \approx R(1) \approx R(J) \quad (23)$$

Those approximations are possible whenever the number of sectors $J$ are large. They allow us to regularize the contour and give rise to overlapping sectors, for which a more robust ML estimator is obtained due to have three times more data. Thus the equation (20) can be rewritten as

$$f(Z/R) = \prod_{j=1}^{J} \prod_{(m,n) \in \epsilon(j)} \left\{ X(m,n)^{\alpha_1(\theta(m,n,\hat{R}(j))) - 1} \right\}^{1/3} \times \frac{1 - X(m,n)^{(\theta(m,n,\hat{R}(j))) - 1}}{B(\alpha_1(\theta(m,n,\hat{R}(j))), \alpha_2(\theta(m,n,\hat{R}(j))))} \quad (24)$$

Since the values of $X(m,n)$ are known, equation (24) is a likelihood function, and our goal is now to find $\hat{R}$ so as to maximize the former. Calculating the negative logarithms we can define the neg-log-likelihood from equation (24) as

$$\frac{1}{3} \sum_{j=1}^{J} \sum_{(m,n) \in \epsilon(j)} \left\{ (1 - \alpha_1(\theta(m,n,\hat{R}(j)))) \ln X(m,n) \right\}$$

$$+ (1 - \alpha_2(\theta(m,n,\hat{R}(j)))) \ln (1 - X(m,n))$$

$$+ \ln B(\alpha_1(\theta(m,n,\hat{R}(j))), \alpha_2(\theta(m,n,\hat{R}(j)))) \quad (25)$$

Since equation (24) is factored into the single contributions of every angular sector, minimization of equation (25) is separable in $j$, thus the closed solution is:

$$\hat{R}(j) = r_k' / k' = \arg \min_k \frac{1}{3} \sum_{(m,n) \in \epsilon(j)} \left\{ (1 - \alpha_1(\theta(m,n,\hat{R}(j)))) \ln X(m,n) \right\}$$

$$+ (1 - \alpha_2(\theta(m,n,\hat{R}(j)))) \ln (1 - X(m,n))$$

$$+ \ln B(\alpha_1(\theta(m,n,\hat{R}(j))), \alpha_2(\theta(m,n,\hat{R}(j)))) \quad (26)$$

5 Hypotheses Verification

The method proposed in the paper has a strong dependence on the prior hypotheses expressed in equations (1) and (2); therefore, a method to test whether our procedure fits in a given scenario is mandatory. This topic is covered in the current section.

For a given image which has just been segmented, the results of the hypotheses test can be considered as a figure of merit of the contour obtained by means of the segmentation procedure described in the last section. For the right performance of the hypotheses tests, the independence of the pixels is very important. The
latter is not completely true for real world images and though this modeling mismatch can be acceptable for segmentation purposes, it is crucial for the hypotheses tests. We obtain a subsampled image for which the pixels are uncorrelated. In order to know the correct decimation rates, we estimate the correlation length, in pixels, in the horizontal and vertical directions, denoted as $d_{o_h}$ and $d_{o_v}$. Thus, for the hypotheses verification, the image will be $I$, instead of $X$. Image $I$ is a decimated version of $X$ with factors $d_{o_h}$ and $d_{o_v}$ in the horizontal and vertical directions respectively. Now, we calculate $\eta_j(\alpha_p)$ and $\sigma^2(\alpha_p)$ using equations (10) and (11) for the decimated image $I$.

We define the following error functions for our decimated image $I$:

$$e^1_{\alpha}(j,p) = \eta_j(\alpha_p) - \eta_j(\alpha_p)$$  
$$e^2_{\alpha}(j,p) = \sigma^2(\alpha_p) - \sigma^2(\alpha_p).$$  

We define the total error values for each sector $j$ as:

$$e^1_j = \sum_{p=1}^{P} [e^1_{\alpha}(j,p)]^2$$  
$$e^2_j = \sum_{p=1}^{P} [e^2_{\alpha}(j,p)]^2.$$

We perform a unilateral significance test on those variables for each sector. If our hypotheses are true, equations (1) and (2) should be satisfied. This means that the two foregoing errors should be small, i.e., they should not exceed a threshold, i.e., the hypotheses are acceptable provided that:

$$e^1_j < \tau_{\eta}$$  
$$e^2_j < \tau_{\sigma^2}.$$  

The determination of the thresholds $\tau_{\eta}$ and $\tau_{\sigma^2}$, has been carried out on an experimental basis so as to obtain a significance level of 5%. We have created several test images of the same size of the decimated image $I$ that satisfied the modeling assumption. The thresholds $\tau_{\eta}$ and $\tau_{\sigma^2}$ have been iteratively corrected to end up having an average rate of success of 95%, using a sample space of 2000 sectors.

6 Experiments

Several ultrasound phantom images have been generated using the Field II simulator [7]. The simulator calculates the ultrasound pressure fields by convolving the spatial impulse response of both the transmitter and the receiver with the transmitted excitation and the tissue response. Figure 3 shows the shape of the images that have been used for the determination of the tissue responses. The phantom image of figure 3(a) has been used to estimate the mean and variance parameters $\eta(\alpha)$ and $\sigma^2(\alpha)$ (see figure 4) from a contour that has been sketched manually. From those parameters and a given center, the image contours of the four images are estimated using our algorithm. The resulting contours are correct as figure 5 shows. The image sizes are $M = 204$ and $N = 260$; the number of sectors is $J = 100$; the number of samples in $\alpha$-domain is $P = 50$; the number of points in each ray is $K = 50$; and the size of the slicing windows for the determination of the mean and variance images is $L = 11$. The results of the hypotheses tests for the four phantom images are shown in table 1. In this table the thresholds and the percentage per sector that the hypotheses tests are passed, are shown. Clearly the images in use pass the hypotheses tests. The correlation lengths are estimated to be $d_{o_h} = 7$, $d_{o_v} = 2$.

Our algorithm, clearly, exhibits a good performance. Using the center of the object as external input, the algorithm is able to recover the contour of the object even though sharp-blurry edges are presented, as long as the hypotheses are satisfied. We have realized experimentally that the segmentation results are fairly insensitive to the position of the object center provided that the number of sectors $J$ is large enough.

7 Conclusions

We have presented a method for extracting the contours of a natural structure based on an image model which encodes important prior knowledge about the
shape of the target object and the image formation process. Our model parameters are estimated from a prototypical image (or set of images) previously labeled by a specialist. A ML estimation procedure is used to extract the contours; due to the model hypotheses, the ML procedure can be decoupled for every sector, and the search is only performed in a discrete space, so the time taken by the algorithm is moderated in a conventional PC.

Further research will explore the use of a MRF model to define a prior probability for the contours to guarantee a more regularized solution.

References


